Tail risk premia and return predictability

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Abstract

The variance risk premium, defined as the difference between the actual and risk-neutral expectations of the forward aggregate market variation, helps predict future market returns. Relying on a new essentially model-free estimation procedure, we show that much of this predictability may be attributed to time variation in the part of the variance risk premium associated with the special compensation demanded by investors for bearing jump tail risk, consistent with the idea that market fears play an important role in understanding the return predictability.

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1. Introduction

The VIX is popularly referred to by market participants as the “investor fear gauge.” Yet, on average, only a small fraction of the VIX is arguably attributable to market fears.
We show that rather than simply buying (selling) when the VIX is high (low), the genuine fear component of the index provides a much better guide for making "good" investment decisions.

Volatility clustering in asset returns is ubiquitous. This widely documented temporal variation in volatility (Schwert, 2011; Andersen, Bollerslev, Christoffersen, and Diebold, 2013) represents an additional source of risk over and above the variation in the actual asset prices themselves. For the market as a whole, this risk is also rewarded by investors, as directly manifest in the form of a wedge between the actual and risk-neutralized expectations of the forward variation of the return on the aggregate market portfolio (Bakshi and Kapadia, 2003). Not only is the variance risk premium on average significantly different from zero, like the variance itself it also fluctuates non-trivially over time (Carr and Wu, 2009; Todorov, 2010). Mounting empirical evidence further suggests that unlike the variance, the variance risk premium is useful for predicting future aggregate market returns over and above the predictability afforded by more traditional predictor variables such as the dividend-price and other valuation ratios, with the predictability especially strong over relatively short quarterly to annual horizons (Bollerslev, Tauchen, and Zhou, 2009).

The main goals of the present paper are twofold. First, explicitly recognizing the prevalence of different types of market risks, we seek to nonparametrically decompose their sum total as embodied in the variance risk premium into separate diffusive and jump risk components with their own distinct economic interpretations. Second, relying on this new decomposition of the variance risk premium, we seek to clarify where the inherent market return predictability is coming from and how it plays out over different return horizons and for different portfolios with different risk exposures.

Extending the long-run risk model of Bansal and Yaron (2004) to allow for time-varying volatility-of-volatility, Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) have previously associated the temporal variation in the variance risk premium with notions of time-varying economic uncertainty. On the other hand, extending the habit formation type preferences of Campbell and Cochrane (1999), Bekaert and Engstrom (2010) and Bekaert, Hoerova, and Lo Ducu (2013) have argued that the variance risk premium may be interpreted as a proxy for aggregate risk-aversion. Meanwhile, as emphasized by Bollerslev and Todorov (2011b), the variance risk premium formally reflects the compensation for two very different types of risks: continuous and discontinuous price moves. The possibility of jumps, in particular, adds an additional unique source of market variance risk stemming from the locally non-predictable nature of jumps. This risk is still present even if the investment opportunity set does not change over time (i.e., even in a static economy with independent and identically distributed returns), and it remains a force over diminishing investment horizons (i.e., even for short time-intervals where the investment opportunity set is approximately constant). As discussed more formally below, these distinctly different roles played by the two types of risks allow us to uniquely identify the part of the variance risk premium attributable to market fears and the special compensation for jump tail risk.

Our estimation of the separate components of the variance risk premium builds on and extends the new econometric procedures recently developed by Bollerslev and Todorov (2014). The basic idea involves identifying the shape of the risk-neutral jump tails from the rate at which the prices of short maturity options decay for successively deeper out-of-the-money contracts. Having identified the shape of the tails, their levels are easily determined by the actual prices of the options. In contrast to virtually all parametric jump-diffusion models hitherto estimated in the literature, which restrict the shape of the tail decay to be constant over time, we show that the shapes of the nonparametrically estimated jump tails vary significantly over time, and that this variation contributes non-trivially to the temporal variation of the variance risk premium. The statistical theory underlying our new estimation procedure is formally based on an increasing cross-section of options. Importantly, this allows for a genuine predictive analysis avoiding the look-ahead bias which invariably plagues other more traditional parametric-based estimation procedures relying on long-span asymptotics for the tail estimation.

The two separately estimated components of the variance risk premium each exhibit their own unique dynamic features. Although both increase during times of financial crisis and distress (e.g., the 1997 Asian crisis, the 1998 Russian default, the 2007–08 global financial crisis, and the 2010 European sovereign debt crisis), the component due to jump risk typically remains elevated for longer periods of time. By contrast, the part of the variance risk premium attributable to “normal” risks rises significantly during other time periods that hardly register in the jump risk component (e.g., the end of the dot-com era in 2002–03). Counter to the implications from popular equilibrium-based asset pricing models, nonparametric regression analysis also suggests that neither of the two components of the variance risk premium can be fully explained as nonlinear functions of the aggregate market volatility.

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4 Following the classical Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973), variance risk has traditionally been associated with changes in the investment opportunity set, which in turn induce a hedging component in the asset demands.


6 The overall level of the market volatility also tends to mean revert more quickly than the jump risk premia following all of these events.

7 The habit persistence model of Campbell and Cochrane (1999), for example, and its extension in Du (2010), imply such a nonlinear relationship.
Hence, nonlinearity of the pricing kernel cannot be the sole explanation for the previously documented predictability inherent in the variance risk premium.\footnote{Similarly, nonlinearity cannot explain the empirically weak mean-variance tradeoff widely documented in the literature; see, e.g., Bollerslev, Sizova, and Tauchen (2012) and the many references therein.}

The distinctly different dynamic dependencies in the two components of the variance risk premium also naturally suggest that the return predictability for the aggregate market portfolio afforded by the total variance risk premium may be enhanced by separately considering the two components in the return predictability regressions. Our empirical results confirm this conjecture. In particular, we find that most of the predictability for the aggregate market portfolio previously ascribed to the variance risk premium stems from the jump tail risk component, and that this component drives out most of the predictability stemming from the part of the variance risk premium associated with “normal” sized price fluctuations. Replicating the predictability regressions for the aggregate market portfolio for size, value, and momentum portfolios comprised of stocks sorted on the basis of their market capitalizations, book-to-market values, and past annual returns, we document even greater increases in the degree of return predictability by separately considering the two variance risk premium components. The predictability patterns for the corresponding zero-cost high-minus-low arbitrage portfolios are generally also supportive of our interpretation of the jump tail risk component of the variance risk premium as providing a proxy for market fears.

Our empirical findings pertaining to the predictability of the aggregate market portfolio are related to other recent empirical studies, which argue that various options-based measures of jump risk are useful for forecasting future market returns. Santa-Clara and Yan (2010), in particular, find that an estimate of the equity risk premium due to jumps, as implied from options and a one-factor stochastic volatility jump-diffusion model, significantly predict subsequent market returns. Similarly, Andersen, Fusari, and Todorov (2015), relying on a richer multifactor specification, find that a factor directly related to the risk-neutral jump intensity helps forecast future market returns. Allowing for both volatility jumps and self-exciting jump intensities, Li and Zinna (2014) report that the predictive performance of the variance risk premium estimated within their model may be improved by separately considering the estimated jump component. All of these studies, however, rely on specific model structures and long time-span asymptotics for parameter estimation and extraction of the state variables that drive the jump and stochastic volatility processes. By contrast, our empirical investigations are distinctly nonparametric in nature, thus imbuing our findings with a built-in robustness against model misspecification.\footnote{A plethora of competing parametric models have been used in the empirical option pricing literature. For instance, while one-factor models, as in, e.g., Pan (2002), Broadie, Chernov, and Johannes (2007), and Santa-Clara and Yan (2010), are quite common, the empirical evidence in Bates (2000) and Christoffersen, Heston, and Jacobs (2009), among others, clearly suggests that multiple volatility factors are needed. Correspondingly, in models that do allow for jumps, the jump arrival rates are typically taken to be constant, although the estimates in Christoffersen, Jacobs, and Ornthanalai (2012) and Andersen, Fusari, and Todorov (2015), suggest that the return predictability for the aggregate market portfolio afford by the total variance risk premium may be enhanced by separately considering the two components in the return predictability regressions.}

Moreover, our approach for estimating the temporal variation in the jump tail risk measures is based on the cross-section of options at a given point-in-time, thus circumventing the usual concerns about structural-stability and “look-ahead” biases that invariably plague conventional parametric-based procedures.

Other related nonparametric-based approaches include Vilkov and Xiao (2013), who argue that a conditional Value-at-Risk (VaR) type measure extracted from options through the use of Extreme Value Theory (EVT) predicts future market returns, although the predictability documented in that study is confined to relatively short weekly horizons. Also, Du and Kapadia (2012) find that a tail index measure for jumps defined as the difference between the sum of squared log-returns and the square of summed log-returns affords some additional predictability for the market portfolio over and above that of the variance risk premium. In contrast to these studies, the new nonparametric jump risk measures proposed and analyzed here are all economically motivated, with direct analogs in popular equilibrium consumption-based asset pricing models. Moreover, the predictability results for the market portfolio and the interpretation thereof are further corroborated by our new empirical findings pertaining to other portfolio sorts and priced risk factors.

The rest of the paper is organized as follows. Section 2 presents our formal setup and definitions of the variance risk premium and its separate components. We also discuss how the jump tail risk component manifests within two popular stylized equilibrium setups. Section 3 outlines our new estimation strategy for nonparametrically extracting the jump tails. Section 4 describes the data that we use in our empirical analysis. The actual estimation results for the new jump tail risk measures is discussed in Section 5. Section 6 presents the results from the return predictability regressions, beginning with the aggregate market portfolio followed by the results for the different portfolio sorts and systematic risk factors. Section 7 concludes.

2. General setup and assumptions

The continuous-time dynamic framework, and corresponding variation measures, underlying our empirical investigations is very general. It encompasses almost all parametric asset pricing models hitherto used in the literature as special cases.

2.1. Returns and variance risk premium

Let $X_t$ denote the price of some risky asset defined on the filtered probability space $(\Omega, \mathcal{F}, P)$, where $(\mathcal{F}_t)_{t \geq 0}$ refers to the filtration. We will assume the following dynamic continuous-time representation for the
instantaneous arithmetic return on $X$,
\[ dX_t \bigg/ X_{t-} = a_t \, dt + \sigma_t \, dW_t + \int_{\mathbb{R}} \left( e^x - 1 \right) \tilde{\mu}^\mathcal{P}(dt, dx), \tag{2.1} \]
where the drift and diffusive processes, $a_t$ and $\sigma_t$, respectively, are both assumed to have càdlàg paths, but otherwise left unspecified, $W_t$ is a standard Brownian motion, and $\mu(dt, dx)$ is a counting measure for the jumps in $X$ with compensator $dt \otimes \nu_t^\mathcal{P}(dx)$, so that $\tilde{\mu}^\mathcal{P}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^\mathcal{P}(dx)$ is a martingale measure under $\mathcal{P}$.\(^\text{10}\)

The continuously compounded return from time to $t + \tau$, say $r_{[t, t+\tau]} \equiv \log(X_{t+\tau}) - \log(X_t)$, implied by the formula in (2.1) may be expressed as,
\[ r_{[t, t+\tau]} = \int_t^{t+\tau} (a_s + q_s) \, ds + \int_t^{t+\tau} \sigma_s \, dW_s + \int_t^{t+\tau} \tilde{\mu}_s^\mathcal{P}(ds, dx), \tag{2.2} \]
where $q_t$ represents the standard convexity adjustment term associated with the transformation from arithmetic to logarithmic returns. Correspondingly, the variability of the price over the $[t, t+\tau]$ time-interval is naturally measured by the quadratic variation,
\[ QV_{[t, t+\tau]} = \int_t^{t+\tau} \sigma_s^2 \, ds + \int_t^{t+\tau} \int_{\mathbb{R}} \nu^\mathcal{P}(dx, ds, dx). \tag{2.3} \]

Even though the diffusive price increments associated with $\sigma$ and the jumps controlled by the counting measure $\mu$ both contribute to the total variation of returns and the pricing thereof, they do so in distinctly different ways.

In order to more formally investigate the separate pricing of the diffusive and jump components, we will assume the existence of the alternative risk-neutral probability measure $\mathcal{Q}$, under which the dynamics of $X$ takes the form,
\[ dX_t \bigg/ X_{t-} = (r_{\delta t} - \delta_t) \, dt + \sigma_t \, dW_t^\mathcal{Q} + \int_{\mathbb{R}} \left( e^x - 1 \right) \tilde{\mu}_t^\mathcal{Q}(dt, dx), \tag{2.4} \]
where $r_{\delta t}$ and $\delta_t$ refer to the instantaneous risk-free rate and the dividend yield, respectively, $W_t^\mathcal{Q}$ is a Brownian motion under $\mathcal{Q}$, and $\tilde{\mu}_t^\mathcal{Q}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^\mathcal{Q}(dx)$ where $dt \otimes \nu_t^\mathcal{Q}(dx)$ denotes the compensator for the jumps under $\mathcal{Q}$. The existence of $\mathcal{Q}$ follows directly from the lack of arbitrage under mild technical conditions (see, e.g., the discussion in Duffie, 2001). Importantly, while the no-arbitrage condition restricts the diffusive volatility process $\sigma_t$ to be the same under the $\mathcal{P}$ and $\mathcal{Q}$ measures, the lack of arbitrage puts no restrictions on the $dt \otimes \nu_t^\mathcal{Q}(dx)$ jump compensator for the “larger” (in absolute value) sized jumps. In that sense, the two different sources of risk manifest themselves in fundamentally different ways in the pricing of the asset.

Consider the (normalized by horizon) variance risk premium on $X$ defined by,
\[ VRP_{[t, t+\tau]} = \frac{1}{\tau} \left( \mathbb{E}_t^\mathcal{P}(QV_{[t, t+\tau]}) - \mathbb{E}_t^\mathcal{Q}(QV_{[t, t+\tau]}) \right). \tag{2.5} \]

This mirrors the definition of the variance risk premium most commonly used in the options pricing literature (see, e.g., Carr and Wu, 2009), where the difference is also sometimes referred to as a volatility spread (see, e.g., Bakshi and Madan, 2006).\(^\text{11}\)

Let $CV_{[t, t+\tau]} = \int_t^{t+\tau} \sigma_s^2 \, ds$. denote the total continuous variation over the $[t, t+\tau]$ time-interval, and denote the corresponding total predictable jump variation under the $\mathcal{P}$ and $\mathcal{Q}$ probability measures by.\(^\text{12}\)
\[ JVP_{[t, t+\tau]} = \int_t^{t+\tau} \int_{\mathbb{R}} \nu^\mathcal{P}(dx, ds, dx) \quad JVP_{[t, t+\tau]} = \int_t^{t+\tau} \int_{\mathbb{R}} \nu^\mathcal{Q}(dx, ds, dx). \]

The variance risk premium may then be decomposed as,
\[ VRP_{[t, t+\tau]} = \frac{1}{\tau} \left( \mathbb{E}_t^\mathcal{P}(QV_{[t, t+\tau]} - JVP_{[t, t+\tau]} + JVP_{[t, t+\tau]}^\mathcal{Q}) - \mathbb{E}_t^\mathcal{Q}(QV_{[t, t+\tau]} + JVP_{[t, t+\tau]}^\mathcal{Q}) \right) \]
\[ = \frac{1}{\tau} \left[ \left( \mathbb{E}_t^\mathcal{P}(QV_{[t, t+\tau]} - JVP_{[t, t+\tau]} + JVP_{[t, t+\tau]}^\mathcal{Q}) \right) \right] + \left( \frac{1}{\tau} \left( \mathbb{E}_t^\mathcal{Q}(QV_{[t, t+\tau]} - JVP_{[t, t+\tau]} + JVP_{[t, t+\tau]}^\mathcal{Q}) \right) \right). \]

The first parenthesis inside the square brackets on the right-hand-side involves the differences between the $\mathcal{P}$ and $\mathcal{Q}$ expectations of the continuous variation. Analogously, the second parenthesis inside the square brackets involves the differences between the $\mathcal{P}$ and $\mathcal{Q}$ expectations of the same $\mathcal{P}$ jump variation measure. These two terms account for the pricing of the temporal variation in the diffusive risk $\sigma_t$ and the jump intensity process $\nu_t^\mathcal{Q}(dx)$, respectively. For the aggregate market portfolio, these differences in expectations under the $\mathcal{P}$ and $\mathcal{Q}$ measures are naturally associated with investors’ willingness to hedge against changes in the investment opportunity set. By contrast, the very last term on the right-hand-side in the above decomposition involves the difference between the expectations of the objective $\mathcal{P}$ and risk-neutral $\mathcal{Q}$ jump variation measures evaluated under the same $\mathcal{P}$ probability measure $\mathcal{Q}$. As such, this term is effectively purged from the compensation for time-varying jump intensity risk. It has no direct analogue for the diffusive price component, but instead reflects the “special” treatment of jump risk.\(^\text{13}\)

\(^{10}\) This implicitly assumes that $X_t$ does not have fixed times of discontinuities. This assumption is satisfied by virtually all asset pricing models hitherto used in the literature.

\(^{11}\) This difference also corresponds directly to the expected payoff on a (long) variance swap contact. Empirically, the variance risk premium for the aggregate market portfolio as defined in (2.5) is on average negative. In the discussion of the empirical results below we will refer to our estimate of $-VRP_{t, t+\tau}$, as the variance risk premium for short.\(^\text{12}\)

\(^{13}\) Formally, the total quadratic variation in (2.3) may alternatively be expressed as,
\[ QV_{[t, t+\tau]} = \log(X_t, log(X_t)_{[t, t+\tau]} + \int_t^{t+\tau} \int_{\mathbb{R}} \nu^\mathcal{P}(dx, ds, dx), \]
where the first term on the right-hand-side corresponds to the so-called predictable quadratic variation, and the second term is a martingale; see, e.g.,Protter (2004). The first predictable quadratic variation term
Without additional parametric assumptions about the underlying model structure it is generally impossible to empirically identify and estimate the separate diffusive and jump risk components.\textsuperscript{14} However, by focusing on the jump “tails” of the distribution, it is possible (under very weak additional semi-nonparametric assumptions) to estimate a measure that parallels the second term in the above decomposition and the part of the variance risk premium due to the special compensation for jump tail risk. Moreover, as we argue in the next section, this new measure may be interpreted as a proxy for investor fears.\textsuperscript{15}

2.2. Jump tail risk

The general dynamic representations in (2.1) and (2.4) do not formally distinguish between different sized jumps. However, there is ample anecdotal as well as more rigorous empirical evidence that “large” sized jumps, or tail events, are viewed very differently by investors than more “normal” sized price fluctuations (see, e.g., Bansal and Shaliastovich, 2011 and the references therein). Motivated by this observation, we will focus on the pricing of unusually “large” sized jumps, with the notion of “large” defined in a relative sense compared to the current level of risk in the economy.\textsuperscript{16} Empirically, of course, without an explicit parametric model it would also be impossible to separately identify the “small” jump moves from the diffusive price increments.

Specifically, define the left and right jump-neutral jump tail variation over the $[t, t+\tau]$ time-interval by,

$$\begin{align*}
LJV^Q_{[t, t+\tau]} &= \int_t^{t+\tau} \int_{x < -k_T} x^2 \nu^Q_t(dx) \, ds, \\
RJV^Q_{[t, t+\tau]} &= \int_t^{t+\tau} \int_{x > k_T} x^2 \nu^Q_t(dx) \, ds,
\end{align*}$$

(2.6)

\textit{Footnote continued}

$captures$ $the$ $risk$ $associated$ $with$ $the$ $temporal$ $variation$ $in$ $the$ $stochastic$ $volatility$ $and$ $its$ $analogue$ $for$ $the$ $jumps;$ $i.e.$ $,$ $the$ $jump$ $intensity$ $\nu^Q_t(dx).$ $The$ $second$ $martingale$ $term$ $associated$ $with$ $the$ $compensated,$ $or$ $demeaned,$ $jump$ $process$ $\mu^Q_t(dx) = \mu(dx) - dt \otimes \nu^Q_t(dx)$ $stems$ $solely$ $from$ $the$ $fact$ $that$ $jumps,$ $or$ $price$ $discontinuities,$ $may$ $occur.$ $This$ $term$ $has$ $no$ $analogue$ $for$ $the$ $diffusive$ $price$ $component.$ $The$ $“special”$ $compensation$ $for$ $jumps$ $refers$ $to$ $the$ $compensated$ $to$ $this$ $second$ $term.$ $In$ $theory,$ $all$ $jumps,$ “small” $and$ “large,” $will$ $contribute$ $to$ $this$ $term.$ $Empirically,$ $however,$ $with$ $discretely$ $sampled$ $prices$ $and$ $options$ $data,$ $it$ $is$ $impossible$ $to$ $uniquely$ $identify$ $and$ $distinguish$ $the$ “small” $jumps$ $from$ $continuous$ $price$ $moves.$ $Hence,$ $in$ $our$ $empirical$ $investigations,$ $we$ $restrict$ $our$ $attention$ $to$ $the$ “special” $compensation$ $for$ $jump$ $tail$ $risk.$

\textsuperscript{14} Andersen, Fusari, and Todorov (2015) have recently estimated the separate components based on a standard two-factor stochastic volatility model augmented with a third latent time-varying jump intensity factor.\textsuperscript{15} Intuitively, for $t \to 0$,

$$\lim_{t \to 0} VRPt_t = \int_R x^2 (\nu^Q_t(dx) - \nu^Q_0(dx)),$$

\textit{corresponding to the second term on the right-hand-side in the decomposition of $VRPt_t,$ and the lack of compensation for changes in the investment opportunity set over diminishing horizons.} \textit{That is, our definition of what constitute “large” sized jumps and our jump tail risk measures are relative as opposed to absolute concepts.}

where $k_T > 0$ is a time-varying cutoff pertaining to the log-jump size.\textsuperscript{17} Let the corresponding left and right jump tail variation measures under the actual probability measure $\mathcal{P}$, say $LJV^\mathcal{P}_{[t, t+\tau]}$ and $RJV^\mathcal{P}_{[t, t+\tau]}$, be defined analogously from the $dt \otimes \nu^\mathcal{P}_t(dx)$ jump tail compensator. In parallel to the definition of the variance risk premium in (2.5), the (normalized by horizon) left and right jump tail risk premia are then naturally defined by,

$$\begin{align*}
LJP_t = \frac{1}{\tau} \left( \mathbb{E}_t^\mathcal{P} (LJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (LJV^\mathcal{Q}_{[t, t+\tau]}) \right), \\
RJP_t = \frac{1}{\tau} \left( \mathbb{E}_t^\mathcal{P} (RJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (RJV^\mathcal{Q}_{[t, t+\tau]}) \right),
\end{align*}$$

(2.7)

both of which contribute to $VRPt_t$. Correspondingly, the difference $VRPt_t - (LJP_t + RJP_t)$ may be interpreted as the part of the variance risk premium attributable to “normal” sized price fluctuations.

Mimicking the decomposition of the variance risk premium discussed in the previous section, the left and right jump premia defined above may be decomposed as,

$$\begin{align*}
LJP_t &= \frac{1}{\tau} \left[ \mathbb{E}_t^\mathcal{P} (LJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (LJV^\mathcal{Q}_{[t, t+\tau]}) \right] \\
&\quad + \frac{1}{\tau} \left[ \mathbb{E}_t^\mathcal{P} (LJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (LJV^\mathcal{Q}_{[t, t+\tau]}) \right],
\end{align*}$$

and

$$\begin{align*}
RJP_t &= \frac{1}{\tau} \left[ \mathbb{E}_t^\mathcal{P} (RJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (RJV^\mathcal{Q}_{[t, t+\tau]}) \right] \\
&\quad + \frac{1}{\tau} \left[ \mathbb{E}_t^\mathcal{P} (RJV^\mathcal{P}_{[t, t+\tau]}) - \mathbb{E}_t (RJV^\mathcal{Q}_{[t, t+\tau]}) \right],
\end{align*}$$

(2.8)

respectively. The first term on the right-hand-side in each of the two expressions involves the difference between the $\mathcal{P}$ and $\mathcal{Q}$ expectations of the same jump variation measures. Again, this directly mirrors the part of the variance risk premium associated with the difference between the $\mathcal{P}$ and $\mathcal{Q}$ expectations of the future diffusive risk $CV^\mathcal{Q}_{[t, t+\tau]}$. By contrast, the second term on the right-hand-side in each of the two expressions involves the difference between the expectations of the respective $\mathcal{P}$ and $\mathcal{Q}$ jump tail variation measures under the same probability measure $\mathcal{Q}$, reflecting the “special” treatment of jump tail risk.\textsuperscript{18}

Under the additional assumption that $\mathcal{P}$ jump intensity process is approximately symmetric for “large” sized jumps, we have $LJV^\mathcal{P}_{[t, t+\tau]} \approx RJV^\mathcal{P}_{[t, t+\tau]}$. Hence, the first terms on the right-hand-sides in the above decompositions of $LJP_t$ and $RJP_t$ will be approximately the

\textsuperscript{17} The use of a time-varying cutoff $k_T$ for identifying the “large” jumps directly mirrors the use of a time-varying threshold linked to the diffusive volatility $\sigma_t$ in the tests for jumps based on high-frequency intraday data pioneered by Mancini (2001).

\textsuperscript{18} In parallel to the expression for the variance risk premium above, it follows that for $t \to 0$,

$$\begin{align*}
\lim_{t \to 0} LJP_t &= \int_{x < -k_T} x^2 (\nu^Q_t(dx) - \nu^Q_0(dx)), \\
\lim_{t \to 0} RJP_t &= \int_{x > k_T} x^2 (\nu^Q_t(dx) - \nu^Q_0(dx)),
\end{align*}$$

\textit{corresponding to the second term on the right-hand-side in the respective decompositions.}
same.\textsuperscript{19} Therefore, for sufficiently large values of the cutoff \(k_t\), the difference between the two jump tail premia,
\[
LJP_{t,t} - RJP_{t,t} = \frac{1}{\tau} \left( E^Q \left( LJV^P_{[t,t+\tau]} \right) - E^Q \left( LJV^Q_{[t,t+\tau]} \right) \right)
\]
will be largelyvoid of the compensation for temporal variation in jump intensity risk. As such, \(LJP_{t,t} - RJP_{t,t}\) may be interpreted as a proxy for investor fears. This mirrors the arguments behind the investor fear index proposed by Bollerslev and Todorov (2011b).\textsuperscript{20} However, in contrast to the estimates reported in Bollerslev and Todorov (2011b), which restrict the shape of the jump tails to be time-invariant, we explicitly allow for empirically more realistic time-varying tail shape parameters, relying on the information in the cross-section of options for identifying the temporal variation in the \(Q\) jump tails.

Going one step further, it follows readily that for approximately symmetric \(P\) jump tails,
\[
LJP_{t,t} - RJP_{t,t} = \frac{1}{\tau} E^Q \left( RJV^Q_{[t,t+\tau]} \right) - \frac{1}{\tau} E^Q \left( RJV^P_{[t,t+\tau]} \right).
\]

thus expressing the fear component of the tail risk premia as a function of the \(Q\) jump tails alone.\textsuperscript{21} As such, this conveniently avoids any tail estimation under \(P\), which inevitably is plagued by a dearth of "large" sized jumps and a "law-of-small-numbers," or Pesos-type problem. Moreover, for the aggregate market portfolio the magnitude of the risk-neutral left jump tail dwarfs that of the right jump tail, so that empirically, \(LJP_{t,t} - RJP_{t,t}\) is approximately equal to the \(Q\) expectation of the negative left jump variation only,
\[
LJP_{t,t} - RJP_{t,t} \approx \frac{1}{\tau} E^Q \left( LJV^Q_{[t,t+\tau]} \right),
\]
affording a particularly simple expression for the fear component.

2.3. Equilibrium interpretations of the jump tail measures

The definition of the jump tail risk premia and their interpretation discussed above hinge solely on the general continuous-time specification for the price process in (2.1) and the corresponding no-arbitrage condition. Importantly, our empirical estimation of the different measures also does not require us to specify any other aspects of the underlying economy. Nonetheless, in order to gain some intuition for the different measures, and \(LJP_{t,t}\) in particular, we briefly consider their manifestation within the context of two popular stylized equilibrium consumption-based asset pricing frameworks.

To begin, we consider a setup built on a representative agent with time non-separable Epstein-Zin preferences and affine dynamics for consumption and dividends. This setup has been analyzed extensively by Eraker and Shallastovich (2008). It includes the long-run risks models of Bansal and Yaron (2004) and Drechsler and Yaron (2011), as well as the rare disaster model with time-varying probabilities for disasters of Gabax (2012) and Wachter (2013) as special cases. In this general setup, the jump intensity under the statistical probability measure \(P\) may be conveniently expressed as,
\[
\nu^P_t(dx) = \left( \nu^P_{t+1} + \cdots + \nu^P_i + \cdots + \nu^P_0 \right) dx,
\]
where \(\nu^P_i\) controls the intensity of different sources of (orthogonal) jumps in the economy (e.g., jumps in consumption growth), which by assumption takes the form,
\[
\nu^P_i(x) = (\alpha \nu^P_i)(x),
\]
for some time-invariant jump intensity measures \(\nu^P_i(x)\) and the \(\nu^P_i\) vector of state variables that drive the dynamics of the fundamentals in the economy. The pricing kernel in this economy in turn implies that the jump intensity process under the risk-neutral probability measure \(Q\) takes the form,
\[
\nu^Q_t(dx) = \left( \nu^Q_{t+1} + \cdots + \nu^Q_i + \cdots + \nu^Q_0 \right) dx,
\]
where
\[
\nu^Q_i(x) = e^{\lambda_t} \nu^P_i(x).
\]
Comparing (2.9) and (2.10) with (2.11) and (2.12), the pricing of all jump risk in this economy is formally based on exponential tilting of the \(P\) jump distribution, with the extent of the tilting and the pricing of the different sources of risks determined by the \(\lambda_t\)'s. The actual values of the \(\lambda_t\)'s will depend on the structural parameters and the risk-aversion of the representative agent in particular. Importantly, the temporal variation in the priced jump risk is driven by the same factors that drive the actual market jump risks.
Further specializing this setup along the lines of the recent rare disaster models of Gabalex (2012) and Wachter (2013) involving a single source of (negative) jumps, the expression for the $Q$ jump intensity simplifies to $
u^Q_t(x) = e^{-\gamma_t x}$, where $\gamma$ refers to the risk-aversion of the representative agent. It follows readily from the definition of $LP_{t,\tau}$, that in this situation,

$$LJP_{t,\tau} = \frac{1}{\tau} \int_t^{t+\tau} \int_{\mathbb{R}} (\nu^P_t(dx)) ds $$

$$\quad + \frac{1}{\tau} \int_t^{t+\tau} \int_{\mathbb{R}} (1 - e^{-\gamma_t x})^2 \nu^Q_t(dx).$$

(2.13)

The second term on the right-hand-side arises solely from the representative agent’s special attitude towards jump risk. Moreover, as this expression shows, any variation in this term is intimately related to the state variables that drive the fundamentals in the economy.

As an alternative equilibrium framework, consider now the generalization of the habit formation model of Campbell and Cochrane (1999) recently proposed by Du (2010), in which the representative agent faces disaster risks in consumption. In this setup, consumption growth is assumed to be independently and identically distributed (i.i.d.) and subject to the possibility of rare disasters in the form of extreme negative jumps, while the agent’s risk-aversion $\gamma_t$ varies with the level of (external) habits determined by aggregate consumption. Correspondingly, the risk-neutral jump intensity may be expressed as,

$$\nu^Q_t(x) = f(\gamma_t) \nu^Q_t(dx),$$

(2.14)

for some nonlinear function $f(\cdot)$. Within this model the pricing of jump risk is therefore directly related to $\gamma_t$ and the pricing of risk in the economy more generally. In contrast to the framework based on an agent with Epstein-Zin preferences, the jump distribution also differs across the different equilibrium settings, it only require that $\nu^Q_t(x)$ satisfies (3.1) for “large” jumps beyond some threshold, say $|x| > \kappa$.

The specification in (3.1) is very general, allowing for two separate sources of independent variation in the jump tails, in the form of “level shifts” governed by $\phi_t^\pm$, and shifts in the rate of decay, or the “shape,” of the tails governed by $\alpha^\pm_t$. By contrast, the assumption of constant tail shape parameters, or $\alpha^+ = \alpha^- = \alpha$, employed in essentially all parametric models estimated in the literature to date imply that the relative importance of differently sized jumps is time-invariant, so that the only way for the intensity of “large” sized jumps to change over time is for the intensity of all sized jumps to change proportionally.22 In most models hitherto employed in the literature that do allow for temporal variation in the jump intensity process $\nu^Q_t(dx)$, it is also assumed that the dynamic dependencies in the left and right tails may be described by the identical level-shift process, with the temporal variation in $\phi_t^\pm = \phi^\pm$ driven by a simple affine function of the diffusive variance $\sigma^2_t$.23

3. Jump tail estimation

Our estimation of the $Q$ jump tail measures builds on the specification for the $\nu^Q_t(dx)$ jump intensity process proposed by Bollerslev and Todorov (2014),

$$\nu^Q_t(dx) = \left( \phi^+_t \times e^{-\alpha^+_t x} \mathbf{1}_{[x > 0]} + \phi^-_t \times e^{-\alpha^-_t |x|} \mathbf{1}_{[x < 0]} \right) dx.$$  

(3.1)

This specification explicitly allows the left (−) and right (+) jump tails to differ. Although it formally imposes the same structure on all sized jumps, the results that follow only require that $\nu^Q_t(x)$ satisfies (3.1) for “large” jumps beyond some threshold, say $|x| > \kappa$.

The specification in (3.1) is very general, allowing for two separate sources of independent variation in the jump tails, in the form of “level shifts” governed by $\phi_t^\pm$, and shifts in the rate of decay, or the “shape,” of the tails governed by $\alpha^\pm_t$. By contrast, the assumption of constant tail shape parameters, or $\alpha^+ = \alpha^- = \alpha$, employed in essentially all parametric models estimated in the literature to date imply that the relative importance of differently sized jumps is time-invariant, so that the only way for the intensity of “large” sized jumps to change over time is for the intensity of all sized jumps to change proportionally.22 In most models hitherto employed in the literature that do allow for temporal variation in the jump intensity process $\nu^Q_t(dx)$, it is also assumed that the dynamic dependencies in the left and right tails may be described by the identical level-shift process, with the temporal variation in $\phi_t^\pm = \phi^\pm$ driven by a simple affine function of the diffusive variance $\sigma^2_t$.23

By contrast, the temporal variation in $\phi_t^\pm$ is left completely unspecified in the present setup.

The jump intensity process in (3.1) readily allows for closed-form solutions for the integrals that define $LJV^Q_{[t,t+\tau]}$ and $RJV^Q_{[t,t+\tau]}$ in Eq. (2.6) in terms of the $\alpha^\pm_t$ and $\phi^\pm_t$ parameters and the cutoff $k_t$ defining “large” jumps. In particular, assuming that the tail parameters remain constant over the horizon $\tau$, the left and right jump tail variation measures may be succinctly expressed as,

$$LJV^Q_{[t,t+\tau]} = \tau \phi^- e^{-\alpha^- k_t} (\alpha^- k_t k_t + 2) / (\alpha^-)^3,$$

$$RJV^Q_{[t,t+\tau]} = \tau \phi^+ e^{-\alpha^+ k_t} (\alpha^+ k_t k_t + 2) / (\alpha^+)^3.$$ 

(3.2)

Our estimation of $\alpha^\pm_t$ and $\phi^\pm_t$, and in turn the $LJV^Q_{[t,t+\tau]}$ and $RJV^Q_{[t,t+\tau]}$ measures, will be based on out-of-the-money (OTM) puts and calls for the left and right tails, respectively. Intuitively, the $\alpha^\pm_t$ parameters may be uniquely identified from the rate at which the prices of the options decay in the tail, while for given tail shapes the $\phi^\pm_t$ parameters may be inferred from the actual option price levels.

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22 This includes the affine jump diffusion models of Duffie, Pan, and Singleton (2000), the time-changed tempered stable models of Carr, Geman, Madan, and Yor (2002), along with the nonparametric estimation procedure employed in Bollerslev and Todorov (2011b).

23 This approach is exemplified by the jump-diffusion models estimated in Pan (2002) and Eraker (2004).
Formally, let \( O_{t,T}(k) \) denote the time \( t \) price of an OTM option on \( X \) with time to expiration \( T \) and log-moneyness \( k \). It follows then from Bollerslev and Todorov (2011b) that for two put options with the same maturity \( t \), but different strikes \( k_1, k_2 \), \( \log(O_{t,T}(k_2)/O_{t,T}(k_1)) \approx (1 + \alpha) (k_2 - k_1) \). Similarly, for two call options with strikes \( k_1, k_2 \), \( \log(O_{t,T}(k_2)/O_{t,T}(k_1)) \approx (1 - \alpha) (k_2 - k_1) \). Utilizing these approximations, Bollerslev and Todorov (2014) show how the time-varying tail shape parameters \( \alpha \pm \) may be consistently estimated from an ever-increasing number of deep OTM short-maturity options by:

\[
\alpha_t^{\pm} = \arg \min_{\alpha^\pm} \frac{1}{N_t^{\pm}} \sum_{i=1}^{N_t^{\pm}} \left| \log \frac{O_{t,T}(k_i)}{O_{t,T}(k_{i-1})} (k_{i-1} - k_{i-1})^{-1} - (1 \pm (-\alpha^\pm)) \right|
\]

where \( N_t^{\pm} \) denotes the total number of calls (puts) used in the estimation with moneyness \( 0 < k_1 < \cdots < k_{N_t^{\pm}} \). (0 < \( k_1 < \cdots < k_{N_t^{\pm}} \). In the results reported on below, we implement this estimator on a weekly basis, thus implicitly assuming that the \( \alpha_t^{\pm} \) parameters only change from week to week.

The estimates for \( \alpha_t^{\pm} \) in (3.3) put no restrictions on the \( \phi_t^{\pm} \) parameters that shift the level of the jump intensity process through time. Meanwhile, let \( \tau_{t,T} \) denote the risk-free interest rate over the \( [t, t + \tau] \) time-interval, and \( F_{t,T} \) the time \( t \) futures price of \( X_{t+\tau} \). It then follows from Bollerslev and Todorov (2014) that for \( t \) and \( k < 0 \), \( e^{\phi_t^{\pm}} O_{t,T}(k)/F_{t+\tau} \approx \tau_{t,T}^{\pm}(\alpha_t^{\pm})/O_{t,T}(k) / F_{t+\tau} = \tau_{t,T}^{\pm}(\alpha_t^{\pm})/O_{t,T}(k) / F_{t+\tau} \). Utilizing these approximations, the “level-shift” parameters may be estimated in a second step by:

\[
\phi_t^{\pm} = \arg \min_{\phi^{\pm}} \frac{1}{N_t} \sum_{i=1}^{N_t} \left| \log \left( \frac{2}{\tau_{t+\tau}} O_{t,T}(k_i) \right) - \left( 1 \mp \alpha_t^{\pm} \right) k_i \right| \]

\[+ \log \left( \alpha_t^{\pm} + 1 \right) + \log \left( \alpha_t^{\pm} \right) - \log \left( \phi^{\pm} \right). \]

Taken together these estimates completely characterize the \( \mathcal{Q} \) jump intensity process in (3.1), and in turn all of the jump tail risk measures defined in Section 2.

4. Data

The data used in our empirical analysis come from three different sources. The raw options data are obtained from OptionMetrics, and consist of closing bid and ask quotes for all S&P 500 options traded on the Chicago Board of Options Exchange (CBOE), along with the corresponding zero coupon rates. The options span the period from January 1996 to August 2013, for a total of 4,445 trading days.

The estimates for the jump tail parameters in (3.3) and (3.4) formally rely on an increasing number of arbitrarily short-lived OTM options to eliminate the impact of the diffusive price component. In an effort to best mimic this condition, we restrict our analysis to options with no more than 45 days until expiration. To help alleviate the impact of market microstructure complications for the shortest-lived options, we also rule out any options with less than eight days to maturity. In practice, of course, for a given fixed maturity, these OTM option prices will still reflect some diffusive risk. To help mitigate this risk, for the estimation of the left jump tail parameters, we only use puts with log-moneyness less than minus two-and-a-half times the maturity-normalized Black-Scholes at-the-money (ATM) implied volatility. Similarly, for the right jump tail parameters, we only use call options with log-moneyness in excess of the maturity-normalized Black-Scholes implied volatility.26 In the end, this leaves us with an average of 100.2 and 51.0 puts and calls per week, respectively, over the full sample.

Our construction of the actual realized variation measures and the variance risk premium rely on high-frequency S&P 500 futures prices obtained from Tick Data Inc. The intraday prices are recorded at five-minute intervals, starting at 8:35 CST until the last price of the day at 15:15 CST, for a total of 81 observations per trading day. We also use these same high-frequency data in testing whether the option-based \( \mathcal{Q} \) jump tail expectations are consistent with the subsequently observed \( \mathcal{P} \) jump tail realizations.

Our aggregate market return predictability regressions are based on a broad value-weighted portfolio of all Center for Research in Security Prices (CRSP) firms incorporated in the U.S. and listed on the NYSE, Amex, or Nasdaq stock exchanges. The relevant time-series of daily returns are obtained from Kenneth R. French’s data library.27 We also rely on that same data source for daily returns on various size, book-to-market, and momentum sorted portfolios. Lastly, we obtain data on the monthly dividend-price ratio for the aggregate market from CRSP.

5. Empirical tail measures

The left and right \( \mathcal{Q} \) jump variation measures introduced above, including the approximate fear component in (2.8), may all be expressed as explicit functions of the jump tail parameters in (3.1). We begin our empirical analysis with a discussion of these parameters and the time-varying left and right “large” jump intensities implied by the estimates.

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24 The use of a robust M-estimator effectively downweights the influence of any “outliers.”

25 Following standard “cleaning” procedures to rule out arbitrage, starting from the closest at-the-money options we omit any out-of-the-money options for which the midquotes do not decrease with the strike price. We also omit any zero bid option prices.

26 By explicitly relating the threshold of the moneyness for the options used in the estimation to the overall level of the volatility, we screen out more relatively close to at-the-money options in periods of high volatility, thereby effectively minimizing the impact of the on average larger diffusive price component in the OTM option price when the volatility is high. Since the market for call options is less liquid than the market for puts, we rely on a more lenient cutoff for the right tail estimation.

27 Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.
5.1. Tail parameters

Our estimates for the weekly left and right jump tail “shape” parameters are based on Eq. (3.3) and all of the qualifying options within each calendar week. The resulting sample mean of $\hat{\alpha}_t$ equals 16.23 compared to 61.81 for $\hat{\alpha}_t^+$, indicative of the on average much slower tail decay inherent in the put versus call OTM option prices. Further to this effect, the top two panels in Fig. 1 show $1/\hat{\alpha}_t^\pm$, corresponding to the left and right jump tail intensities. The estimates for the left tail index vary almost ten-fold over the sample, ranging from a low of around 0.03 in 1997 and 2007, to a high of more than 0.25 in 2008–09 at the height of the recent financial crisis. Although less dramatic, the estimates for the right tail index also exhibit substantial variation over time. These temporal dependencies are directly manifest in the form of first-order autocorrelations for the left and right tail “shape” parameters equal to 0.59 and 0.67, respectively.

The jump intensity process, of course, also depends on the “level” parameters. Our weekly estimates for these are based on the expression in (3.4). Rather than plotting the estimates for $\phi_t^\pm$, the bottom two panels in Fig. 1 show the annualized left and right “large” jump intensities implied by $\hat{\alpha}_t^\pm$ and $\hat{\phi}_t^\pm$.

The calculation of these measures also necessitates a choice for the cutoff $k_t$ pertaining to the log-jump size and the start of the jump “tails.” For both of the plots in the figure, as well as the $LJV_t$ and $LJV_t$ jump variation measures reported on below, we fix $k_t$ at 6.868 times the normalized Black-Scholes ATM volatility at time $t$. This specific cutoff corresponds to the median strike price for the deepest OTM puts in the sample.

Allowing the $\alpha_t^\pm$ tail “shape” parameters to vary over time, results in fairly stable and mildly correlated intensities for the “large” negative jumps. Meanwhile, there is a sense of “euphoria” and relatively high jump intensities for the “large” positive jumps embedded in the OTM call option prices leading up to the financial crisis. Of course, the right jump tail intensities are orders of magnitude less than those for the left jump tail. We turn next to a discussion of the jump tail variation measures and risk premia implied by these estimates for the $\nu_t^O(dx)$ “large” jump intensity process.

5.2. Jump tail variation measures

Our estimates for the weekly left and right $\nu_t^O$ jump variation measures, as implied by Eq. (3.2), are depicted in Fig. 2. Looking first at $LJV_t$ in the top panel, the measure inherits many of the same key dynamic dependencies evident in the left tail index shown in the top left panel in Fig. 1. However, referring to Panel B in Table 1, the sample correlation between $LJV_t$ and $LJ_t$ is only equal to 0.26. By contrast, the correlation between $LJV_t$ and the right tail intensity $RJV_t$ equals 0.89. Of course, as Fig. 2 and Table 1 both make clear, $RJV_t$ is orders of magnitude less than $LJV_t$, so the $\text{fear}$ component defined as the difference between the two is effectively equal to $-LJV_t$, as previously stated in (2.8).

To underscore the importance of explicitly allowing both the “shape” and the “level” of the jump tails to change over time in the estimation of this new $\text{fear}$ component, the left panel in Fig. 3 shows the estimates for the left jump tail variation $LJV_t^n$ obtained by restricting $\alpha_t^- = \alpha_t^-$ to be constant, but allowing $\phi_t^-$ to change over time. Correspondingly, the right panel shows the estimates for $LJV_t^n$ obtained by restricting $\phi_t^- = \phi^-$ to be constant, but allowing $\alpha_t^-$ to be time-varying. Restricting the “shape” parameter to be constant, as is commonly done in the literature, clearly mutes the temporal variation and cuts the sample standard deviation of $LJV_t^n$ in half compared to $LJV_t$. By contrast, restricting the temporal variation to be solely driven by the “shape” of the jump tails, results in an even more dramatic increase in the magnitude of the $\text{fear}$ component during the recent financial crisis. Along these lines, it is also worth noting that the first-order sample autocorrelation for $LJV_t$ is larger than the autocorrelations of both $LJV_t^n$ and $LJV_t^n$. Consistent with the return predictability results discussed below, $LJV_t$ also correlates more strongly with $LJV_t^n$ than $LJV_t^n$.

The stylized equilibrium models discussed in Section 2.3 imply that the variation in $LJV_t$, is a direct, possibly nonlinear, function of the spot volatility. To investigate this conjecture empirically, Fig. 4 presents the results from a nonparametric kernel regression of our nonparametric estimate of $LJV_t$ on the at-the-money implied variance from the shortest-maturity options available on the day (with at least eight days to maturity), where the latter serves as a proxy for the unobservable spot volatility.

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28 Our finding of time-varying $\alpha_t^\pm$ parameters is consistent with the evidence for serially correlated “extreme” returns based on the so-called extremogram estimator in Davis and Mikosch (2000) and Davis, Mikosch, and Cribben (2012). The recent cross-sectional-based tail index estimates reported in Chollete and Lu (2011), Kelly and Jiang (2014), and Ruenzi and Weigert (2011) also point to strong dynamic dependencies. All of these studies, however, pertain to the actual return distributions and the shape of the tails under $P$. Recent studies that have estimated somewhat simpler dynamic dependencies in the tails under $Q$ include Almeida, Vicente, and Guillen (2013), Du and Kapadia (2012), Hamidieh (2011), Sirivardane (2013), and Vilkov and Xiao (2013).

29 We also experimented with other choices for this “tail” cutoff, resulting in qualitatively very similar dynamic features and predictability regressions to the ones reported below. Further details concerning these additional results are available in a Supplementary Appendix.

30 The correlation between $LJV_t$ and the $\text{fear index}$ estimated in Bollerslev and Todorov (2011b) relying on long-span asymptotics and the more restrictive assumption of constant tail “shape” parameters equals 0.75.

31 The reported kernel density estimates are based on a Gaussian kernel with the bandwidth parameter set according to the prescription in Bowman and Azzalini (1997). We also experimented with the use of alternative nonparametric estimates for the spot volatility obtained from high-frequency data on the S&P 500 index futures, resulting in very similar nonparametric regression estimates for $LJV_t$.

32 As the time to maturity converges to zero, the at-the-money implied volatility formally converges to the diffusive spot volatility; see, e.g., Durrleman (2008).
shows, there is a substantial amount of variation in $LJV_t$ that cannot be explained by the current market volatility, even when allowing for a highly nonlinear relation between the two series. Further, as directly seen from the right panel in Fig. 4, forcing $LJV_t$ to have the same value for a given market volatility produces a fitted variation measure with a much more pronounced spike than the actual $LJV_t$ series in the aftermath of the dot-com bubble and the mild economic recession in the early 2000s. On the other hand, since the spot volatility is generally faster mean-reverting than the actual $LJV_t$ series, the nonlinear projection of $LJV_t$ on the volatility series results in a shorter-lived impact of the recent financial crises. In sum, $LJV_t$ contains its own unique dynamic dependencies, which cannot be spanned by the volatility.

5.3. Jump tail variation and return correlations

The sample correlations between the weekly returns on the aggregate market portfolio $MRK$ and the different jump tail variation measures, reported in the first row in Panel B of Table 1, are all negative. This mirrors the contemporaneous asymmetric return-volatility relationship, or so-called “leverage effect,” widely documented in the literature for other volatility measures and models; see, e.g., the discussion in Bollerslev, Sizova, and Tauchen (2012) and the references therein. At the same time, the contemporaneous correlations between the different tail variation measures and the weekly returns on the small minus big (SMB), high minus low (HML) and winners minus losers (WML) zero-cost portfolios, further analyzed below, are all smaller (in absolute value) and some even positive.

Meanwhile, with the exceptions of $RJV_t$, the sample correlations between the jump tail variation measures and the market return over the subsequent week, reported in the first row in Panel C of Table 1, are all positive. This suggests that a risk-return tradeoff, or “volatility feedback effect,” may also be operative, whereby an increase (decrease) in one of the variation measures causes an immediate drop (rise) in the price in order to allow for higher (lower) future returns as a compensation for the increased (decreased) risk. Of course, these unconditional sample correlations do not distinguish whether the higher (lower) returns are indeed associated with an increase in systematic risk or a change in the attitude towards risk, or both.

5.4. Tail “shape” variation: risk or attitude to risk?

Our interpretation of $LJV_t$ as a measure of market fears hinges on the standard no-arbitrage condition and the fact that it does not restrict the form of the $dt \otimes \nu^2_t(dx)$ jump compensator for the “large” sized jumps in (2.4) vis-a-vis the $dt \otimes \nu^2_t(dx)$ jump compensator in (2.1). If, on the other hand, jumps and diffusive price moves were treated as
identical risks by investors, the jump intensity process should be the same under the $P$ and $Q$ measures. Consequently, the mapping from $\nu_t^L(dx)$ to $\nu_t^R(dx)$ directly reflects the “special” compensation for jump tail risk in the economy, as exemplified by the exponential tilting in Eq. (2.12) implied by the stylized long-run risk and rare disaster models, or the proportional shift from $P$ to $Q$ in Eq. (2.14) implied by the habit formation model.

The estimation of a general process for $\nu_t^L(dx)$ that parallels that of $\nu_t^R(dx)$ in (3.1) is inevitably plagued by a dearth of “large” jump tail realizations over short weekly time intervals. Instead, as a way to meaningfully test whether the “shape” of the risk-neutral and actual jump tails are indeed the same, as implied for example by the habit persistence model with measure change given in (2.14), we consider the time-series of actual high-frequency-based tail realizations over the full sample. In particular, let $\eta_t$ denote the threshold for defining the “large” negative jump realizations. Provided the jump tail shapes are identical, the martingale condition is satisfied.

\[
\int_t^{t+\tau} \int_{x < -\eta_t} \left[ |x| - \frac{(1+\alpha^L_t \eta_t)}{\alpha^L_t} \right] \mu(ds, dx),
\]

should then be a martingale under the statistical probability measure $P$.\footnote{An analogous martingale condition obviously holds for the right jump tails.} Importantly, this condition does not depend on the overall “level” of the actual jump tails, but only on their “shape.”

Substituting the weekly estimates of $\hat{\alpha}^L_t$ for the $Q$ jump tails in place of $\alpha^L_t$, the above martingale condition is readily operationalized as a test for identical jump tail “shapes” under the $P$ and $Q$ measures. Specifically, define the vector of sample moments,

\[
\tilde{m} = \frac{1}{T} \left( \sum_{s \in [0, T]} \log(X_s) \right) \left( \frac{1+\hat{\alpha}^L_s \eta_s}{\hat{\alpha}^L_s} \right) \otimes \psi_s,
\]

where $\psi_s$ refers to any vector of valid instruments. Also, let $W$ denote an estimate for the corresponding asymptotic variance-covariance matrix for $\tilde{m}$.\footnote{By standard arguments, the covariance matrix may be consistently estimated by,

\[
\hat{W} = \frac{1}{T} \left( \sum_{s \in [0, T]} \left[ \log(X_s) \right] \left( \frac{1+\hat{\alpha}^L_s \eta_s}{\hat{\alpha}^L_s} \right)^2 \otimes \psi_s \right).
\]}

If the martingale condition is satisfied, $\tilde{m}^T W^{-1} \tilde{m}$ should be (asymptotically for increasing sample size $T$) distributed as a Chi-square distribution with degrees of freedom equal to the dimension of the instrument vector $\psi_s$.

In implementing the test we rely on high-frequency intraday five-minute returns along with a time-varying threshold for determining the “large” sized jumps.\footnote{The thresholds that we use explicitly adjust for the temporal variation in the daily continuous volatility based on the realized bipower variation over the previous day as well as the strong intraday volatility pattern based on an estimate of the time-of-day effect; for additional details, see Bollerslev and Todorov (2011b).} In particular, fixing $\eta_t$ at six times the estimated continuous five-minute return variation, together with a two-dimensional instrument vector comprised of a constant...
Table 1
Summary statistics.
The table reports summary statistics for weekly returns, estimated jump tail parameters, and jump tail variation measures. The data range from January 1996 to August 2013. The variation measures are recorded at the end-of-the-week, with the returns spanning the corresponding Monday close-to-close. The returns are in weekly percentage form. All of the variation measures are in annualized percentage form, except for the right tail variation measure which is in annualized percentage squared form.

Panel A: Univariate statistics

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<td>0.02</td>
<td>0.10</td>
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Panel B: Contemporaneous correlations

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<th>SML</th>
<th>HML</th>
<th>WML</th>
<th>α−</th>
<th>α+</th>
<th>LJI</th>
<th>RJII</th>
<th>LJV</th>
<th>RJV</th>
<th>LJV*</th>
<th>LJV**</th>
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<td>1.00</td>
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<td>0.42</td>
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<td>1.00</td>
<td>0.04</td>
<td>0.42</td>
<td>0.72</td>
</tr>
<tr>
<td>RJV</td>
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<td>0.42</td>
<td>0.72</td>
<td>1.00</td>
<td>0.04</td>
<td>0.42</td>
<td>0.72</td>
<td>1.00</td>
<td>0.04</td>
<td>0.42</td>
<td>0.72</td>
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<tr>
<td>LJV*</td>
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<td>0.06</td>
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<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
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<td>LJV**</td>
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Panel C: One-week-ahead return correlations

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<th>RJII</th>
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<th>LJV**</th>
</tr>
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<td>MRK</td>
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<td>−0.03</td>
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<td>−0.01</td>
<td>0.05</td>
<td>−0.00</td>
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</tr>
<tr>
<td>SML</td>
<td>−0.00</td>
<td>−0.05</td>
<td>−0.02</td>
<td>−0.02</td>
<td>−0.12</td>
<td>−0.03</td>
<td>−0.09</td>
<td>−0.06</td>
</tr>
<tr>
<td>HML</td>
<td>−0.02</td>
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<td>−0.05</td>
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<td>0.06</td>
<td>−0.04</td>
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</tbody>
</table>

Fig. 3. Left jump variation with restricted tail parameters. The left panel plots the left jump variation $LJV_\text{t}$ obtained by restricting the “shape” parameter $\alpha^−$ to be constant, but allowing $\phi^−$ to change over time. The right panel shows $LJV_\text{t}^\ast$ obtained by restricting the “level” parameter $\phi^−$ to be constant, but allowing $\alpha^−_\text{t}$ to be time-varying. The estimates are reported in annualized squared form.
and an estimate of the integrated volatility over the previous day relative to the occurrence of a jump at time \( s \), results in a test statistic of 175.6, thus strongly rejecting the null hypothesis that diffusive and jump risks are treated the same by investors.37

At a general level, the test therefore also supports the idea that \( LJ_V \) affords a “cleaner” proxy for market fears than the variance risk premium, let alone the popularly used \( VIX \) “investor fear gauge.” To further explore this, we turn next to the results from a series of standard monthly-based return predictability regressions for return horizons ranging up to a year, using different variation measures as explanatory variables.

6. Return predictability regressions

Several recent studies have argued for the existence of a statistically significant link between the variance risk premium and future returns on the aggregate market portfolio, with this predictive relationship especially strong over three- to six-month horizons (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011; Du and Kapadia, 2012; Bekaert and Hoerova, 2014; Bollerslev, Marrone, Xu, and Zhou, 2014; Camponovo, Scaillet, and Trojani, 2013; Vilkov and Xiao, 2013, among several other studies). The results from our predictability regressions complement and expand on these findings by explicitly considering the new jump tail variation measures, and the part of the variance risk premium due to jump tail risk, as separate predictor variables.

Following standard practice in the literature, we rely on a monthly observation frequency for all of our return predictability regressions. Specifically, let \( R_{[t,t+\tau]} \) denote the continuously compounded return from time \( t \) to \( t+\tau \) formally defined in Eq. (2.2) above, with the unit time interval corresponding to a month. The return regressions discussed below may then be expressed as,

\[
R_{[t,t+h]} = \alpha_t + \beta_t V_t + u_{t,t+h}, \quad t = 1, 2, \ldots, T-h, \tag{6.1}
\]

where \( V_t \) refers to one or more of the variation measures and other explanatory variables, and the horizon range from \( h=1 \) (one-month) to \( h=12 \) (one year). To account for the overlap that occurs for \( h > 1 \), we rely on the standard robust Newey-West \( t \)-statistics with a lag length equal to two times the return horizon. In addition, to help alleviate concerns about size distortions and over-rejections known to plague inference in overlapping return regressions with persistent predictor variables (see, e.g., Ang and Bekaert, 2007), following Hodrick (1992) we also report robust \( t \)-statistics for the null of no predictability from the reverse regressions.38 Further, to aid with the interpretation of the results from the multiple regressions involving more than one explanatory variable, we report Newey-West and Hodrick Wald-tests for the null of no predictability by any of the predictor variables included in the regression.39 We turn next to a discussion of the specific explanatory variables considered in the return predictability regressions.

6.1. Variation measures and other explanatory variables

Our estimation of the jump tail parameters \( \hat{\alpha}_t \pm \) and \( \hat{\phi}_t \pm \), and the resulting new jump tail variation measures discussed above, closely follows the approach developed in Bollerslev and Todorov (2014) and the choice of a weekly estimation frequency advocated therein. Even though this results in

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37 This particular choice of threshold results in a total of 285 left tail jumps over the full sample. We also experimented with other choices of \( \eta \), resulting in equally strong rejections. The corresponding test for the right jump tails equals 87.0, which also strongly rejects the null of identical jump tail “shapes” under the \( P \) and \( Q \) measures.

38 As forcefully emphasized by Hodrick (1992), the Hodrick-based \( t \)-statistics are only formally valid under the null of no predictability, not just by the regressor in question but by any predictor variable. As such, they are difficult to interpret even in simple regressions when more than one explanatory variable is useful for predicting the returns, let alone in multiple regressions. By contrast, the Newey-West \( t \)-statistics are always (asymptotically) justified and interpretable.

39 With two predictor variables, the 5% and 1% critical values in the corresponding asymptotic Chi-square distribution with two degrees of freedom equal 5.991 and 9.210, respectively.
unbiased parameter estimates, the estimates are invariably somewhat noisy. To help smooth out this estimation error, and directly match the observation frequency of the jump tail variation measures to the observation frequency of the return regressions, in the empirical results reported on below we rely on the monthly variation measures obtained by averaging the within-month weekly values. Comparing the plot for the resulting monthly LJVt series given in the top panel in Fig. 5 to the weekly series shown in the top panel in Fig. 2 clearly shows the effect of smoothing out the estimation error, and the implicit use of a coarser monthly estimation frequency. Still, the weekly and monthly series obviously share the same general features and dynamic dependencies.

The second panel in Fig. 5 plots the monthly VIXt series. The VIX, of course, represents an approximation to the risk-neutral expectation of the total quadratic variation in \( \Delta t \), and as such reflects the compensation for both time-varying diffusive volatility and jump intensity risks, as well as market expectations about future jump tail events and the “special” pricing thereof. Meanwhile, comparing our \( LJVt \) fear proxy to the VIXt reveals a strong coherence between the two series. At the same time, however, there are also some important differences. In particular, even though both series attained their maximum in-sample values around October 2008, the VIX fairly quickly reverted to pre-crisis levels, while LJVt remained elevated for a much longer period of time. LJVt also experienced another more prolonged period of elevated jump tail risk in 2010 in connection with the European sovereign debt crisis. Similarly, LJVt remained unusually high over a longer period of time than VIXt in the aftermath of the East Asian crises starting in July 1997 and the August 1998 Russian default. Conversely, the collapse of the tech bubble and the declining equity valuations in 2002 clearly resulted in higher values of the VIX, but hardly affected the left jump variation.\(^{41}\)

The third panel in Fig. 5 shows the variance risk premium minus the left jump tail variation \( VRP_t - LJV_t \), or the part of the variance risk premium attributable to “normal” sized price fluctuations.\(^{42}\) Our estimate of the variance risk premium is based on the difference between the VIXt and the expectation of the forward variation aggregated over the month, as derived from the same multivariate forecasting model for the high-frequency-based realized variation measures developed in Bollerslev and Todorov (2011b).\(^{43}\)

Interestingly, the figure reveals a much less dramatic impact of the recent financial crisis on VRPt – LJVt than on LJVt and VIXt. On the other hand, VRPt – LJVt was relatively high for part of 2002–03, while this period hardly registered for the left jump tail variation measure. These differences in the variation measures also directly manifest in the return predictability regressions discussed below.

As previously noted, the log dividend–price ratio arguably represents the most widely used predictor variable in the literature. Although, it is commonly thought that the dividend–price ratio offers the most predictability over longer multi-year horizons, the results in Ang and Bekaert (2007) suggest that the predictability is actually the strongest over shorter within-year horizons as analyzed here. In parallel to the variation-based measures, the log dividend–price ratio \( \log(D_t / P_t) \) shown in the bottom panel in Fig. 5 also increased quite dramatically during the recent financial crisis and the accompanying precipitous drop in equity valuations. Meanwhile, comparing the plot of \( \log(D_t / P_t) \) to the plots of the variation-based measures in the first three panels reveals a much more smoothly evolving series void of most of the other clearly discernible peaks associated with other readily identifiable economic and financial events.\(^{44}\)

The next section presents the results from the return predictability regressions based on our new LJVt measure and these other explanatory variables. We begin by discussing the results for the aggregate market portfolio MRK.

### 6.2. Aggregate market return

As noted above, the predictability afforded by the variance risk premium appears to be the strongest over intermediate three- to six-month return horizons. Focusing on the six-month horizon, the left panel in Table 2 reports the results from simple predictability regressions with standard Newey-West \( t \)-statistics in parentheses and Hodrick \( t \)-statistics in square brackets. The sixth column, in particular, confirms the existing empirical evidence that a higher (lower) variance risk premium tends to be associated with higher (lower) returns over the next six months. Meanwhile, comparing the results to the regression reported in the first column, the degree of predictability inherent in VRPt is dominated by that of the LJVt left jump tail variation measure. Further to this effect, subtracting LJVt from VRPt results in less significant \( t \)-statistics for VRPt – LJVt, and lowers the \( R^2 \) relative to the regression based on VRPt, by a factor of one-half.

The regressions for the left jump variation measures \( LJV^*_t \) and \( LJV^{**}_t \) that restrict either \( \alpha_t \) or \( \phi_t \) to be constant show that allowing for temporal variation in both the “shape” and the “level” of the jump tails enhances the predictability. Indeed, the regression coefficient associated with the \( LJV^*_t \) jump tail variation measure extracted under the conventional assumption of time-invariant tail “shapes” is insignificant.\(^{45}\)

\(^{40}\) For reasons of symmetry, we apply the same monthly averaging to all of the variation measures used as explanatory variables in the return regressions. We also performed predictive regressions where VRP and VIX were not averaged over the month with the results being very similar to the ones reported below for their monthly-averaged counterparts.

\(^{41}\) These differences between the VIX and LJV mirror the differences between the option-based estimates for the time-varying diffusive and jump intensity risks in the fully parametric three-factor stochastic volatility model recently estimated by Andersen, Fusari, and Todorov (2015).

\(^{42}\) Following a number of recent studies in the empirical asset pricing literature, we will refer to – VRP, as the variance risk premium, and correspondingly, VRP – LJV, as the part of the premium due to “normal” sized price moves. This, of course, is immaterial for the fit of the return predictability regressions.

\(^{43}\) Consistent with the idea that on average it is profitable to sell volatility, the sample mean of VRPt equals 1.20 in annualized percentage form.

\(^{44}\) The first-order autocorrelation for each of the four monthly series shown in Fig. 5 equal 0.67, 0.79, 0.53, and 0.96, respectively.

\(^{45}\) The plots of \( 1/\alpha_t \) and \( LJV_t \) previously discussed in Fig. 1 also suggest that most of the discernible variation in LJV, resides in the “shape” as opposed to the intensity of the “large” negative jumps.
Also, the right jump variation measure $RJV_t$ does not help in predicting the future returns. The results from the multiple regressions reported in the right panel of the table further corroborate these findings. Including either $RJV_t$ or $VRP_t - LJV_t$ in a multiple regression together with $LJV_t$, leaves only $LJV_t$ significant.\footnote{Including $LJV_t$ and $VRP_t$ in the same regression, obviously results in the same $R^2$ as the regressions based on $LJV_t$ and $VRP_t - LJV_t$. The Newey-West $t$-statistics for $VRP_t$ and $VRP_t - LJV_t$ are also the same, while the $t$-statistics for $LJV_t$ drop from 4.39 for the regression reported in Table 2 to 3.66 for the regression based on $LJV_t$ and $VRP_t$.}

Altogether, this suggests that much of the return predictability previously ascribed to the variance risk premium is effectively coming from the part of the premium due to the left jump tail variation. Further along these lines, it is worth noting that replacing $LJV_t$ with the nonparametric kernel density estimate thereof discussed in Section 5.2, reduces the $R^2$ from 6.54 to 4.54. Moreover, when including both the fitted value of $LJV_t$ and the corresponding residual from the nonparametric regression as explanatory variables in the same return predictability regression, both remain significant with Newey-West $t$-statistics of 2.17 and 2.13, respectively, underscoring the fact that $LJV_t$ and the predictability therein cannot be spanned by the market spot volatility.

As an additional robustness check, we also include the log dividend–price ratio as a possible predictor variable. In line with the existing literature (see, e.g., Ang and Bekaert, 2007 and the many additional references therein), the $t$-statistics for $\log(D_t/P_t)$ in the simple regression reported in the last column in the first part of Table 2 are highly significant. Consistent with the results for the return horizons in excess of three months reported in Bollerslev, Tauchen, and Zhou (2009), the $R^2$ of 18.9 also far exceeds that of the variance risk premium and the other return variation measures. Meanwhile, including $\log(D_t/P_t)$ and $LJV_t$ in the same multiple regression
Table 2
Market return predictability regressions.

The table reports the estimated regression coefficients and \( R^2 \)'s from return predictability regressions for the six-month excess return on the aggregate market portfolio \( \text{MRK} \). The returns are observed monthly with the sample period ranging from January 1996 to August 2013. Newey-West and Hodrick \( t \)-statistics and \( \chi^2(2) \) Wald tests for the significance of the estimated regression coefficients accounting for the overlap in the regressions are reported in parentheses and square brackets, respectively.

<table>
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<tr>
<th></th>
<th>( \text{LJV} )</th>
<th>( \text{VRP} )</th>
<th>( \text{LJV}^* )</th>
<th>( \text{VRP}^* )</th>
<th>( \text{VRP} - \text{LJV} )</th>
<th>( \log(D/P) )</th>
<th>( R^2 )</th>
<th>Wald</th>
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<tr>
<td>( \text{Constant} )</td>
<td>-0.749</td>
<td>2.756</td>
<td>-0.755</td>
<td>-0.237</td>
<td>-0.172</td>
<td>0.906</td>
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<td>( R\text{JV} )</td>
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<td>[3.943]</td>
<td>[8.418]</td>
<td>[2.960]</td>
<td>[2.738]</td>
<td>[3.128]</td>
<td>[1.919]</td>
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<td>( R\text{VJR} )</td>
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<td>[1.155]</td>
<td>[2.618]</td>
<td>[1.836]</td>
<td>[1.245]</td>
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<tr>
<td>( \text{VRP} )</td>
<td>1.343</td>
<td>[2.618]</td>
<td>[2.618]</td>
<td>[2.618]</td>
<td>[2.618]</td>
<td>[2.618]</td>
<td>[2.618]</td>
<td>[2.618]</td>
</tr>
<tr>
<td>( \text{VRP} - \text{LJV} )</td>
<td>0.624</td>
<td>(3.891)</td>
<td>[1.896]</td>
<td>[2.943]</td>
<td>[3.237]</td>
<td>[3.128]</td>
<td>[3.128]</td>
<td>[3.128]</td>
</tr>
<tr>
<td>( \log(D/P) )</td>
<td>25.74</td>
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<td>[3.330]</td>
<td>[1.919]</td>
<td>[0.442]</td>
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<td>[1.919]</td>
<td>[1.919]</td>
</tr>
<tr>
<td>( R^2 )</td>
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<td>7.194</td>
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<tr>
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<td>[1.836]</td>
<td>[2.378]</td>
<td>[2.378]</td>
<td>[2.378]</td>
<td>[2.378]</td>
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</tr>
</tbody>
</table>

Table 3
Market return predictability regressions at different horizons.

The table reports one- to 12-month return predictability regressions for the aggregate market portfolio, analogous to the six-month regressions reported in Table 2.

<table>
<thead>
<tr>
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<th>Three months</th>
<th>Nine months</th>
<th>12 months</th>
</tr>
</thead>
<tbody>
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<td>( \text{Constant} )</td>
<td>-0.049</td>
<td>0.308</td>
<td>-0.226</td>
<td>-0.163</td>
</tr>
<tr>
<td>( \text{VRP} - \text{LJV} )</td>
<td>0.329</td>
<td>0.294</td>
<td>0.949</td>
<td>0.851</td>
</tr>
<tr>
<td>Wald</td>
<td>(8.498)</td>
<td>(17.61)</td>
<td>(12.36)</td>
<td>(14.19)</td>
</tr>
</tbody>
</table>

The predictability in the variance risk premium and its jump tail component. In particular, while \( \text{VRP} - \text{LJV} \) is significant in all of the simple regressions, except for the one-month horizon, the simple regressions based on \( \text{LJV} \) generally result in larger \( t \)-statistics and the \( R^2 \)'s are also much higher.47

47 To put the monthly \( R^2 \)'s reported in the table into perspective, Huang, Jiang, Tu, and Zhou (2015) have recently shown that a properly aligned version of the investor sentiment index originally proposed by Baker and Wurgler (2006) results in a statistically and economically

regression further increases the \( R^2 \) to 21.4, and even though the \( t \)-statistics for \( \log(D_t/P_t) \) and \( \text{LJV} \), are reduced somewhat compared to the two simple regressions, the Wald tests for their joint significance are highly significant. As such, this supports the idea that the dividend yield is not able to fully span the predictive information in the \( \text{LJV} \) tail variation measure. This, of course, is also the case for the stylized theoretical equilibrium models discussed in Section 2.3.

Turning to Table 3 and the regression results for other return horizons reveals the same general patterns vis-a-vis the predictability in the variance risk premium and its jump tail component. In particular, while \( \text{VRP} - \text{LJV} \) is significant in all of the simple regressions, except for the one-month horizon, the simple regressions based on \( \text{LJV} \) generally result in larger \( t \)-statistics and the \( R^2 \)'s are also much higher.47
plots of the corresponding Newey-West $t$-statistics and adjusted regression $R^2$'s for all of the one- through 12-month return regressions shown in Fig. 6 further illustrate this. The $t$-statistics from the simple regressions based on $LJV_t$ are all significant, and the $R^2$'s increase with the return horizons. On the other hand, the $R^2$'s from the simple regressions based on $VRP_t - LJV_t$ plateau at the four-month horizon, while the $R^2$'s from the multiple regressions based on both predictor variables are fairly close to those from the simple regressions based on $LJV_t$ only.

The general setup and discussion in Section 2, along with the test in Section 5.4, suggest that the $VRP_t - LJV_t$ predictor variable may be interpreted as a measure of economic uncertainty, while $LJV_t$ is more readily associated with notions of market fears and the special compensation for jump tail risk. To further buttress this interpretation and help explain where the predictability is coming from, we turn next to a series of predictability regressions for various portfolio sorts and the three common Fama-French-Carhart risk factors.

6.3. Portfolio sorts and risk factors

Different portfolios may respond differently to changes in risk and risk-aversion depending on their risk exposures. To this end, Table 4 reports the results from the same multiple regressions based on $LJV_t$ and $VRP_t - LJV_t$, reported in Tables 2 and 3, replacing the aggregate market portfolio with portfolios comprised of stocks sorted according to their market capitalization, book-to-market (B/M) value, and most recent annual return. For considerations of space, we focus our discussion on the six equally weighted portfolios made up of the top and bottom quintiles for each of the three different sorts.

Beginning with the results pertaining to the size-sorted portfolios reported in the first two columns of Table 4, a clear distinction emerges in the influences of $LJV_t$ and $VRP_t - LJV_t$. In particular, while $VRP_t - LJV_t$ is not as significant as $LJV_t$ for the aggregate market portfolio, it is the most significant predictor for the small-stock portfolio. The regression $R^2$'s for the small-stock portfolio reach an impressive 19.3% at the annual level. Meanwhile, the results for the large-stock portfolio more closely mirror those for the aggregate market, with the predictability primarily driven by $LJV_t$.

To purge the returns from systematic market risks, the third column reports the regression results from the Small-Minus-Big (SMB) zero-cost portfolio defined by the returns on the previously discussed small minus large-stock portfolios. In contrast to the aggregate market portfolio, the longer-run predictability for this portfolio is exclusively coming from the $VRP_t - LJV_t$ variation measure. The results from the simple regressions for the SMB portfolio shown in Fig. 7 further underscore this point. While the $t$-statistics associated with $VRP_t - LJV_t$ for predicting the three- through 12-month returns are all significant at conventional levels, the $t$-statistics for $LJV_t$
Table 4
Portfolio return predictability regressions.

The table reports the results from one- to 12-month return predictability regressions for size (20% smallest and biggest firms), book-to-market (20% highest and lowest B/M ratios), and momentum (20% top and bottom performers over the past 2–12 months) portfolios, along with the corresponding zero-cost portfolios, analogous to the results for the aggregate market in Tables 2 and 3.

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Big</th>
<th>SMB</th>
<th>High</th>
<th>Low</th>
<th>HML</th>
<th>Winners</th>
<th>Losers</th>
<th>WML</th>
</tr>
</thead>
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<tr>
<td><strong>One-month returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.569</td>
<td>0.012</td>
<td>0.557</td>
<td>0.601</td>
<td>-0.180</td>
<td>0.781</td>
<td>1.132</td>
<td>-2.278</td>
<td>3.410</td>
</tr>
<tr>
<td>L/V</td>
<td>(0.980)</td>
<td>(0.024)</td>
<td>(1.462)</td>
<td>(1.126)</td>
<td>-0.368</td>
<td>(2.395)</td>
<td>(1.163)</td>
<td>(1.273)</td>
<td>(2.369)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>(0.027)</td>
<td>1.179</td>
<td>-1.151</td>
<td>0.260</td>
<td>1.430</td>
<td>-1.169</td>
<td>1.281</td>
<td>4.327</td>
<td>-3.045</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>(0.037)</td>
<td>(2.666)</td>
<td>-2.040</td>
<td>(0.463)</td>
<td>(3.244)</td>
<td>-2.761</td>
<td>(1.337)</td>
<td>(1.606)</td>
<td>(1.180)</td>
</tr>
<tr>
<td>R²</td>
<td>0.027</td>
<td>1.179</td>
<td>3.617</td>
<td>0.260</td>
<td>1.430</td>
<td>-1.169</td>
<td>1.281</td>
<td>4.327</td>
<td>-3.045</td>
</tr>
<tr>
<td>Wald</td>
<td>(1.744)</td>
<td>(1.163)</td>
<td>(0.960)</td>
<td>(1.321)</td>
<td>(1.792)</td>
<td>-0.425</td>
<td>(0.601)</td>
<td>(2.060)</td>
<td>(1.692)</td>
</tr>
<tr>
<td><strong>Three-month returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.546</td>
<td>0.174</td>
<td>0.373</td>
<td>1.459</td>
<td>-0.548</td>
<td>2.008</td>
<td>3.656</td>
<td>-10.59</td>
<td>14.25</td>
</tr>
<tr>
<td>L/V</td>
<td>(0.362)</td>
<td>(0.133)</td>
<td>(0.381)</td>
<td>(1.001)</td>
<td>-0.420</td>
<td>(1.918)</td>
<td>(1.448)</td>
<td>(2.019)</td>
<td>(3.389)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>(0.144)</td>
<td>[-0.141]</td>
<td>(0.451)</td>
<td>(0.765)</td>
<td>-0.724</td>
<td>(2.106)</td>
<td>(1.054)</td>
<td>(2.003)</td>
<td>(3.157)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>(0.144)</td>
<td>(1.636)</td>
<td>(0.606)</td>
<td>(2.341)</td>
<td>(2.042)</td>
<td>(1.926)</td>
<td>(3.707)</td>
<td>(3.740)</td>
<td>(3.387)</td>
</tr>
<tr>
<td>R²</td>
<td>1.134</td>
<td>2.449</td>
<td>1.780</td>
<td>0.900</td>
<td>3.867</td>
<td>2.551</td>
<td>0.521</td>
<td>4.518</td>
<td>4.156</td>
</tr>
<tr>
<td>Wald</td>
<td>(3.042)</td>
<td>(7.795)</td>
<td>(7.592)</td>
<td>(1.865)</td>
<td>(13.73)</td>
<td>(8.388)</td>
<td>(1.842)</td>
<td>(5.066)</td>
<td>(2.871)</td>
</tr>
<tr>
<td><strong>Six-month returns</strong></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.339</td>
<td>0.357</td>
<td>-0.696</td>
<td>2.078</td>
<td>-0.522</td>
<td>2.600</td>
<td>5.871</td>
<td>-23.27</td>
<td>29.14</td>
</tr>
<tr>
<td>L/V</td>
<td>-0.112</td>
<td>(0.127)</td>
<td>-0.386</td>
<td>(0.614)</td>
<td>-0.200</td>
<td>(1.127)</td>
<td>(1.222)</td>
<td>(2.085)</td>
<td>(3.680)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>-0.516</td>
<td>[-0.522]</td>
<td>-0.164</td>
<td>(0.188)</td>
<td>-0.993</td>
<td>(1.367)</td>
<td>(0.555)</td>
<td>(2.189)</td>
<td>(2.902)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>(0.940)</td>
<td>(2.323)</td>
<td>[-1.238]</td>
<td>(0.991)</td>
<td>(2.762)</td>
<td>-2.046</td>
<td>(1.104)</td>
<td>(2.420)</td>
<td>(1.894)</td>
</tr>
<tr>
<td>Wald</td>
<td>(2.663)</td>
<td>(3.708)</td>
<td>(0.167)</td>
<td>(1.772)</td>
<td>(4.723)</td>
<td>-1.303</td>
<td>(1.833)</td>
<td>(3.669)</td>
<td>(2.578)</td>
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<tr>
<td><strong>Nine-month returns</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.015</td>
<td>1.107</td>
<td>-1.092</td>
<td>3.275</td>
<td>0.137</td>
<td>3.138</td>
<td>8.196</td>
<td>-27.825</td>
<td>36.021</td>
</tr>
<tr>
<td>L/V</td>
<td>(0.003)</td>
<td>(0.240)</td>
<td>-0.368</td>
<td>(0.617)</td>
<td>(0.032)</td>
<td>(0.877)</td>
<td>(1.030)</td>
<td>(1.903)</td>
<td>(4.201)</td>
</tr>
<tr>
<td>VRP – L/V</td>
<td>[-0.512]</td>
<td>[-0.605]</td>
<td>[-0.076]</td>
<td>(0.016)</td>
<td>[-0.963]</td>
<td>(1.155)</td>
<td>(0.185)</td>
<td>(1.951)</td>
<td>(2.798)</td>
</tr>
</tbody>
</table>
are all insignificant. Moreover, the \( R^2 \)'s from the multiple regressions that include both \( VRP_t - LJV_t \) and \( LJV_t \), shown in the bottom panel of the figure, are very close to the \( R^2 \)'s from the simple regressions based on \( VRP_t - LJV_t \) only.

The \( SMB \) portfolio has previously been associated with variables that describe changes in the investment opportunity set (e.g., Petkova, 2006). As such, our finding that the predictability of the \( SMB \) portfolio is solely driven by \( VRP_t - LJV_t \) is consistent with the idea that this measure is most directly associated with notions of economic uncertainty. Along these lines, a number of studies have also argued that smaller firms tend to be more strongly affected by credit market conditions than larger firms and therefore also more susceptible to general economic conditions (see, e.g., Perez-Quiros and Timmermann, 2000), thus further helping to explain why \( VRP_t - LJV_t \) is a better predictor for the return on the small-stock portfolio.

The results for the value and growth portfolios along with the corresponding High-Minus-Low (\( HML \)) book-to-market portfolio reported in the next three columns in Table 4 convey a very different picture. The predictability pattern for the low book-to-market portfolio, in particular, fairly closely mirrors that of the aggregate market portfolio. On the other hand, the \( R^2 \)'s for the \( HML \) portfolio in Fig. 8 appear to be maximized at the intermediate four-month horizon, with all of the predictability attributable to the \( LJV \) jump tail variation measure.

The returns on portfolios comprised of growth stocks have previously been related to measures of funding liquidity risk (e.g., Asness, Moskowitz, and Pedersen, 2013). Thus, to the extent that liquidity conditions reflect market sentiment, the results in Fig. 8 for the \( HML \) portfolio again indirectly corroborate the interpretation of \( LJV \), as a measure of market fears.

Turning to the results for the momentum portfolios, reported in the last three columns in Table 4, they reveal some very high \( R^2 \)'s for the “loser” portfolio made up of the stocks that performed the worst over the previous 12 months. As shown in Fig. 9, this also translates into very impressive \( R^2 \)'s for the Winners-Minus-Losers (\( WML \)) portfolio in excess of 30% at the six- to eight-month horizons. In contrast to the predictability of the \( SMB \) portfolio, which appears to be driven solely by \( VRP_t - LJV_t \), and the predictability of \( HML \), which is solely attributable to \( LJV \), both of the two variation measures contribute to the predictability of the \( WML \) portfolio.

The return on momentum portfolios, and “loser” portfolios in particular, have also previously been associated with funding liquidity risk (e.g., Pastor and Stambaugh, 2003; Korajczyk and Sadka, 2004). The returns on momentum portfolios have also been shown to exhibit option-like characteristics and be predictable by the “state” of the economy and the volatility of the aggregate market (e.g., Daniel, Jagannathan, and Kim, 2012; Daniel and Moskowitz, 2014), as well as the magnitude of the volatility risk premium (e.g., Nagel, 2012; Fan, Imerman, and Dai, 2013). Consistent with these earlier findings, the much stronger predictability for the “loser” and \( WML \) portfolios based on the decomposition of \( VRP \), reported here again support the interpretation of \( VRP_t - LJV_t \) and \( LJV_t \) as
Fig. 7. *SMB* return predictability regressions. The top panel shows the Newey-West t-statistics from simple return predictability regressions for the Small Minus Big (*SMB*) market capitalization sorted zero-cost portfolio based on the left jump tail variation *LJV* (solid line) and the difference between the variance risk premium and the left jump variation *VRP*−*LJV* (dashed line). The bottom panel shows the corresponding *R*²’s, along with the *R*²’s from multiple regressions including both *LJV* and *VRP*−*LJV* (dotted line).

Fig. 8. *HML* return predictability regressions. The top panel shows the Newey-West t-statistics from simple return predictability regressions for the High Minus Low (*HML*) book-to-market sorted zero-cost portfolio based on the left jump tail variation *LJV* (solid line) and the difference between the variance risk premium and the left jump variation *VRP*−*LJV* (dashed line). The bottom panel shows the corresponding *R*²’s, along with the *R*²’s from multiple regressions including both *LJV* and *VRP*−*LJV* (dotted line).
separate proxies for market uncertainty and fears, respectively, both of which help predict the momentum returns.

7. Conclusion

The variance risk premium, defined as the difference between the actual and risk-neutral expectations of the forward variation of the aggregate market portfolio, is naturally decomposed into two fundamentally different sources of market variance risk: “normal” sized price fluctuations and jump tail risk. We argue that the part of the variance risk premium associated with the compensation for left jump tail risk, in particular, may be seen as a proxy for market fears. We develop new procedures for empirically estimating this component of the variance risk premium from the options surface at a given point in time. Consistent with the idea that this jump risk component represents separate state variables that drive the market risk premium, we find that the explanatory power of return predictability regressions based on the total variance risk premium, as previously reported in the literature, may be significantly enhanced by including the jump tail risk component as a separate predictor variable. Our empirical findings corroborate the theoretical equilibrium-based interpretation of the variance risk premium components and the econometric procedures underlying their separate estimation are both essentially model-free, a more structural-based estimation might afford a deeper understanding of the economic mechanisms that drive the temporal variation in the separate components and help shed new light on the validity of competing equilibrium-based asset pricing models. We leave further work along these lines for future research.

References


